

Relations between Neutrino and Charged Fermion Masses

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We find an intriguing relation between neutrino and charged fermion masses, $|m_{\nu_3}^2 - m_{\nu_1}^2| : (m_{\nu_2}^2 - m_{\nu_1}^2) :: V_{tb}^4 m_\tau^2 m_b^2 / m_t^2 : V_{cs}^4 m_\mu^2 m_s^2 / m_c^2$. We further indicate this relation can be predicted by a left-right symmetric model.

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The neutrino oscillation data have determined the values of the two neutrino mass squared difference [1],

$$\begin{aligned} \Delta m_{21}^2 &= m_{\nu_2}^2 - m_{\nu_1}^2 = 7.65_{-0.20}^{+0.23} \times 10^{-5} \text{ eV}^2, \\ |\Delta m_{31}^2| &= |m_{\nu_3}^2 - m_{\nu_1}^2| = 2.40_{-0.11}^{+0.12} \times 10^{-3} \text{ eV}^2. \end{aligned} \quad (1)$$

We notice the ratio of the above neutrino mass squared difference can be well described by the quark and charged lepton masses,

$$|\Delta m_{31}^2| : \Delta m_{21}^2 :: \frac{m_b^2}{m_t^2} m_\tau^2 V_{tb}^4 : \frac{m_s^2}{m_c^2} m_\mu^2 V_{cs}^4, \quad (2)$$

with the charged fermion masses [2],

$$\begin{aligned} m_u &= 1.27_{-0.42}^{+0.50} \text{ MeV}, & m_d &= 2.90_{-1.19}^{+1.24} \text{ MeV}, \\ m_c &= 0.619 \pm 0.084 \text{ GeV}, & m_s &= 55_{-15}^{+16} \text{ MeV}, \\ m_t &= 171.7 \pm 3.0 \text{ GeV}, & m_b &= 2.89 \pm 0.09 \text{ GeV}, \\ m_e &= 0.486570161 \pm 0.000000042 \text{ MeV}, \\ m_\mu &= 102.7181359 \pm 0.0000092 \text{ MeV}, \\ m_\tau &= 1776.24_{-0.19}^{+0.20} \text{ MeV}, \end{aligned} \quad (3)$$

and the Cabibbo-Kobayashi-Maskawa (CKM) [3] matrix [4],

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415_{-0.0011}^{+0.0010} \\ 0.00874_{-0.00037}^{+0.00026} & 0.0407 \pm 0.0010 & 0.999133_{-0.000043}^{+0.000044} \end{pmatrix}. \quad (4)$$

Here the charged fermion masses and the CKM matrix are all given at $\mu = m_Z$.

A possible solution to the intriguing relation (2) is to consider the normal hierarchy neutrino masses,

$$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = \epsilon \left(\frac{m_d}{m_u} m_e V_{ud}^2, \frac{m_s}{m_c} m_\mu V_{cs}^2, \frac{m_b}{m_t} m_\tau V_{tb}^2 \right), \quad (5)$$

where the parameter ϵ is defined by

$$\begin{aligned} \epsilon &= \sqrt{\frac{|\Delta m_{31}^2|}{V_{tb}^4 m_\tau^2 m_b^2 / m_t^2 - V_{ud}^4 m_e^2 m_d^2 / m_u^2}} \\ &= \sqrt{\frac{\Delta m_{21}^2}{V_{cs}^4 m_\mu^2 m_s^2 / m_c^2 - V_{ud}^4 m_e^2 m_d^2 / m_u^2}}. \end{aligned} \quad (6)$$

We take

$$\begin{aligned} m_u &= 1.27 \text{ MeV}, \quad m_c = 0.703 \text{ GeV}, \quad m_t = 168.7 \text{ GeV} \\ m_d &= 2.90 \text{ MeV}, \quad m_s = 40 \text{ MeV}, \quad m_b = 2.98 \text{ GeV}, \\ m_e &= 0.486570161 \text{ MeV}, \quad V_{ud} = 0.97419, \\ m_\mu &= 102.7181359 \text{ MeV}, \quad V_{cs} = 0.97334, \\ m_\tau &= 1776.24 \text{ MeV}, \quad V_{tb} = 0.999133, \\ \Delta m_{21}^2 &= 7.60 \times 10^{-5} \text{ eV}^2, \\ \Delta m_{31}^2 &= 2.40 \times 10^{-3} \text{ eV}^2, \end{aligned} \quad (7)$$

to derive

$$\epsilon = 1.60 \times 10^{-9}, \quad (8)$$

and then the normal hierarchy neutrino masses,

$$\begin{aligned} m_{\nu_1} &= 1.69 \times 10^{-3} \text{ eV}, \quad m_{\nu_2} = 8.88 \times 10^{-3} \text{ eV}, \\ m_{\nu_3} &= 4.90 \times 10^{-2} \text{ eV}, \quad \sum_{i=1}^3 m_{\nu_i} = 0.0596 \text{ eV}, \end{aligned} \quad (9)$$

which are consistent with the cosmological limit [5].

We further find the appearance of the relation (2) is not accidental in a $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ left-right symmetric model [6]. In our model, the scalar sector contains a real singlet $\sigma(\mathbf{1}, \mathbf{1}, 0)$ and a leptoquark singlet $\delta(\mathbf{3}, \mathbf{1}, \mathbf{1}, -\frac{2}{3})$ as well as a left-handed doublet $\phi_L(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$ and its right-handed partner $\phi_R(\mathbf{1}, \mathbf{1}, \mathbf{2}, -1)$. In the fermion sector, besides the usual quark and lepton doublets, i.e. $q_L(\mathbf{3}, \mathbf{2}, \mathbf{1}, \frac{1}{3})$, $q_R(\mathbf{3}, \mathbf{1}, \mathbf{2}, \frac{1}{3})$, $l_L(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$ and $l_R(\mathbf{1}, \mathbf{1}, \mathbf{2}, -1)$, we introduce four types of singlets: $D_{L,R}(\mathbf{3}, \mathbf{1}, \mathbf{1}, -\frac{2}{3})$, $U_{L,R}(\mathbf{3}, \mathbf{1}, \mathbf{1}, \frac{4}{3})$, $E_{L,R}(\mathbf{1}, \mathbf{1}, \mathbf{1}, -2)$ and $N_R(\mathbf{1}, \mathbf{1}, \mathbf{1}, 0)$. Here all fermions have three generations and their family indices have been suppressed. The left-right symmetry is assumed to be the charge-conjugation, under which the fields transform as

$$\begin{aligned} \sigma &\leftrightarrow -\sigma, \quad \phi_L \leftrightarrow \phi_R^*, \quad \delta \leftrightarrow \delta^*, \quad q_L \leftrightarrow q_R^c, \quad D_L \leftrightarrow D_R^c, \\ U_L &\leftrightarrow U_R^c, \quad l_L \leftrightarrow l_R^c, \quad E_L \leftrightarrow E_R^c, \quad N_R \leftrightarrow N_R. \end{aligned} \quad (10)$$

Here the charge-conjugation of the fermions is defined by $q_L = P_L q \leftrightarrow q_R^c = (q_R)^c = (P_R q)^c = P_L q^c$, etc. We also impose a $U(1)^3 = U(1)_1 \times U(1)_2 \times U(1)_3$ global symmetry, under which $(q_L, q_R^c, D_L^c, D_R, U_L^c, U_R)$, (l_L, l_R^c, N_R) , (E_L, E_R^c) and δ , respectively, carry the quantum numbers $(1, 0, 1)$, $(0, 1, -1)$, $(2, 1, 1)$ and $(1, 1, 0)$, while σ and $\phi_{L,R}$ are trivial. Clearly, this global symmetry is consistent with the left-right symmetry (10).

The full scalar potential is easy to read,

$$\begin{aligned} V = & \frac{1}{2}\mu_\sigma^2\sigma^2 + \mu_\phi^2(|\phi_L|^2 + |\phi_R|^2) + \mu\sigma(|\phi_L|^2 - |\phi_R|^2) \\ & + \mu_\delta^2|\delta|^2 + \frac{1}{4}\lambda_\sigma\sigma^4 + \lambda_\phi(|\phi_L|^4 + |\phi_R|^4) \\ & + \lambda'_\phi|\phi_L|^2|\phi_R|^2 + \lambda_\delta|\delta|^4 + \lambda_{\sigma\phi}\sigma^2(|\phi_L|^2 + |\phi_R|^2) \\ & + \lambda_{\sigma\delta}\sigma^2|\delta|^2 + \lambda_{\phi\delta}(|\phi_L|^2 + |\phi_R|^2)|\delta|^2. \end{aligned} \quad (11)$$

Here the parity-odd singlet σ is essential to realize the spontaneous D-parity violation [7], which can guarantee the breakdown of $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. As for the Yukawa interactions, only the following terms are allowed,

$$\begin{aligned} \mathcal{L}_Y = & -y_D(\bar{q}_L\tilde{\phi}_L D_R + \bar{q}_R^c\tilde{\phi}_L^* D_L^c) - y_U(\bar{q}_L\phi_L U_R + \bar{q}_R^c\phi_R^* U_L^c) \\ & - y_N(\bar{l}_L\phi_L N_R + \bar{l}_R^c\phi_R^* N_R) - h(\delta\bar{l}_L i\tau_2 q_L^c + \delta^*\bar{l}_R^c i\tau_2 q_R) \\ & - f(\delta\bar{U}_L E_L^c + \delta^*\bar{U}_R^c E_R) + \text{H.c.} \end{aligned} \quad (12)$$

We further introduce the mass terms of the fermion singlets by softly breaking the $U(1)^3$ global symmetry,

$$\begin{aligned} \mathcal{L}_{soft} = & -m_D\bar{D}_L D_R - m_U\bar{U}_L U_R - m_E\bar{E}_L E_R \\ & - \frac{1}{2}m_N\bar{N}_R^c N_R + \text{H.c.}, \end{aligned} \quad (13)$$

where the mass matrices m_D , m_U , m_E and m_N are all symmetric, i.e.,

$$m_D = m_D^T, \quad m_U = m_U^T, \quad m_E = m_E^T, \quad m_N = m_N^T. \quad (14)$$

Without loss of generality and for convenience we will choose the base with the diagonal and real m_D , m_U , m_E and m_N .

We now demonstrate that the quarks, the charged leptons, and the neutrinos will obtain their Dirac masses,

$$\mathcal{L} \supset -\tilde{m}_d\bar{d}_L d_R - \tilde{m}_u\bar{u}_L u_R - \tilde{m}_e\bar{e}_L e_R - \tilde{m}_\nu\bar{\nu}_L \nu_R + \text{H.c.}, \quad (15)$$

after the symmetry breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle\phi_R\rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi_L\rangle} U(1)_{em}$. It is easy to check that the f -terms in the Yukawa couplings (12) will contribute to the fermion masses only through their radiative corrections to the masses of the leptoquark singlet δ and the fermion singlets $U_{L,R}$. Therefore, we will not mention the f -terms in the following calculations and discussions.

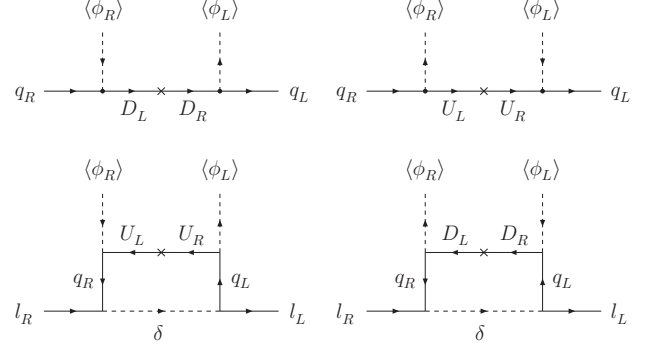


FIG. 1: Diagrams for generating the Dirac masses of the down-type quarks (top left), the up-type quarks (top right), the charged leptons (bottom left) and the neutrinos (bottom right).

As shown in the tree-level diagrams of Fig. 1, the quark masses are induced by integrating out the colored fermion singlets [8],

$$\tilde{m}_d = -y_D \frac{v_L v_R}{m_D} y_D^T = \tilde{m}_d^T, \quad \tilde{m}_u = -y_U \frac{v_L v_R}{m_U} y_U^T = \tilde{m}_u^T, \quad (16)$$

with

$$v_R = \langle\phi_R\rangle \quad \text{and} \quad v_L = \langle\phi_L\rangle \simeq 174 \text{ GeV}. \quad (17)$$

Note that the top quark mass is very close to v_L . We thus need $m_U \sim v_R \gg v_L$ to fulfill the perturbative and seesaw [9] conditions. The choice is similar for m_D . The charged leptons and neutral neutrinos can not acquire the Dirac masses at tree level. Instead, their Dirac masses are generated at one-loop order, as shown in the box diagrams of Fig. 1. We calculate the loop-induced masses to be

$$\tilde{m}_e = \frac{c_e}{16\pi^2} h y_U^* \frac{v_L v_R}{m_U} y_U^\dagger h^T = -\frac{c_e}{16\pi^2} h \tilde{m}_u^\dagger h^T = \tilde{m}_e^T, \quad (18)$$

$$\tilde{m}_\nu = \frac{c_\nu}{16\pi^2} h y_D^* \frac{v_L v_R}{m_D} y_D^\dagger h^T = -\frac{c_\nu}{16\pi^2} h \tilde{m}_d^\dagger h^T = \tilde{m}_\nu^T. \quad (19)$$

Here the coefficients

$$\begin{aligned} c_e = & \ln \frac{m_{U_i}^2 + m_\delta^2}{m_\delta^2} + \frac{m_{U_i}^2}{m_\delta^2} \ln \frac{m_{U_i}^2 + m_\delta^2}{m_{U_i}^2} \\ \simeq & 1 + \ln \frac{m_{U_i}^2}{m_\delta^2} = \mathcal{O}(1-10) \quad \text{for } m_\delta^2 \ll m_{U_i}^2, \end{aligned} \quad (20)$$

$$\begin{aligned} c_\nu = & \ln \frac{m_{D_i}^2 + m_\delta^2}{m_\delta^2} + \frac{m_{D_i}^2}{m_\delta^2} \ln \frac{m_{D_i}^2 + m_\delta^2}{m_{D_i}^2} \\ \simeq & 1 + \ln \frac{m_{D_i}^2}{m_\delta^2} = \mathcal{O}(1-10) \quad \text{for } m_\delta^2 \ll m_{D_i}^2, \end{aligned} \quad (21)$$

can be treated as constants since they are not very sensitive to the running of $m_{U_i}^2/m_\delta^2$ or $m_{D_i}^2/m_\delta^2$. The Dirac

masses between the left- and right-handed neutrinos can be determined by the charged fermion masses, i.e.

$$\tilde{m}_\nu = \frac{c_\nu}{c_e} U_e \sqrt{\hat{m}_e} \frac{1}{\sqrt{\hat{m}_u}} V_{\text{CKM}}^* \hat{m}_d V_{\text{CKM}}^\dagger \frac{1}{\sqrt{\hat{m}_u}} \sqrt{\hat{m}_e} U_e^T. \quad (22)$$

Here $V_{\text{CKM}} = V_u V_d^\dagger$ is the CKM matrix while \hat{m}_d , \hat{m}_u and \hat{m}_e are the diagonal mass matrices of the charged fermions,

$$\begin{aligned} \hat{m}_d &= V_d \tilde{m}_d V_d^T = \text{diag}\{m_d, m_s, m_b\}, \\ \hat{m}_u &= V_u \tilde{m}_u V_u^T = \text{diag}\{m_u, m_c, m_t\}, \\ \hat{m}_e &= U_e^\dagger \tilde{m}_e U_e^* = \text{diag}\{m_e, m_\mu, m_\tau\}. \end{aligned} \quad (23)$$

Note that we require the Yukawa couplings $h \lesssim 1$ for a perturbative theory. With this constraint, the loop-induced charged lepton masses can still arrive at the desired values. Actually, the loop factor is expected to give the mass ratio between the tau lepton and the top quark.

The completed neutrino mass terms are given by

$$\begin{aligned} \mathcal{L} \supset & -\tilde{m}_\nu \bar{\nu}_L \nu_R - y_N v_L \bar{\nu}_L N_R - y_N v_R \bar{\nu}_R^c N_R \\ & - \frac{1}{2} m_N \bar{N}_R^c N_R + \text{H.c.} \\ = & -\frac{1}{2} (\bar{\nu}_L, \bar{\nu}_R^c, \bar{N}_R) \begin{pmatrix} 0 & \tilde{m}_\nu & y_N v_L \\ \tilde{m}_\nu & 0 & y_N v_R \\ y_N^T v_L & y_N^T v_R & m_N \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ N_R \end{pmatrix} \\ & + \text{H.c.} \end{aligned} \quad (24)$$

For \tilde{m}_ν and $y_N v_L$ much smaller than $y_N v_R$ and/or m_N , we can make use of the seesaw [9] formula,

$$\mathcal{L} \supset -\frac{1}{2} m_\nu \bar{\nu}_L \nu_L^c + \text{H.c.}, \quad (25)$$

where the mass matrix m_ν contains two parts,

$$m_\nu = \tilde{m}_\nu \frac{1}{y_N^T} m_N \frac{1}{y_N} \tilde{m}_\nu \frac{1}{v_R^2} - 2 \tilde{m}_\nu \frac{v_L}{v_R}. \quad (26)$$

The first term is the double [10] or inverse [11] seesaw whereas the second one is the linear [12] seesaw. We further assume

$$\frac{1}{y_N^T} m_N \frac{1}{y_N} \frac{1}{v_R^2} = 2 \frac{v_L}{v_R} \left(\frac{1}{\tilde{m}_\nu} + \frac{1}{\tilde{m}_\nu} U_1 \tilde{m}_\nu U_1^T \frac{1}{\tilde{m}_\nu} \right), \quad (27)$$

to parametrize the neutrino mass matrix (26),

$$m_\nu = 2 \frac{v_L}{v_R} U_1 \tilde{m}_\nu U_1^T. \quad (28)$$

Consequently, we can perform

$$\begin{aligned} m_\nu &= 2 \frac{v_L}{v_R} \frac{c_\nu}{c_e} (U_1 U_e) \sqrt{\hat{m}_e} \frac{1}{\sqrt{\hat{m}_u}} V_{\text{CKM}}^* \hat{m}_d V_{\text{CKM}}^\dagger \\ &\times \frac{1}{\sqrt{\hat{m}_u}} \sqrt{\hat{m}_e} (U_e^T U_1^T). \end{aligned} \quad (29)$$

With the charged fermion masses (3) and the CKM matrix (4), it is easy to find

$$\begin{aligned} & \sqrt{\hat{m}_e} \frac{1}{\sqrt{\hat{m}_u}} V_{\text{CKM}}^* \hat{m}_d V_{\text{CKM}}^\dagger \frac{1}{\sqrt{\hat{m}_u}} \sqrt{\hat{m}_e} \\ \simeq & \begin{pmatrix} \frac{m_e}{m_u} [m_d (V_{ud}^*)^2 + m_s (V_{us}^*)^2 + m_b (V_{ub}^*)^2] & \sqrt{\frac{m_e m_\mu}{m_u m_c}} m_s V_{us}^* V_{cs}^* & \sqrt{\frac{m_e m_\tau}{m_u m_t}} m_b V_{ub}^* V_{tb}^* \\ \sqrt{\frac{m_e m_\mu}{m_u m_c}} m_s V_{us}^* V_{cs}^* & \frac{m_\mu}{m_c} [m_s (V_{cs}^*)^2 + m_b (V_{cb}^*)^2] & \sqrt{\frac{m_\mu m_\tau}{m_c m_t}} m_b V_{cb}^* V_{tb}^* \\ \sqrt{\frac{m_e m_\tau}{m_u m_t}} m_b V_{ub}^* V_{tb}^* & \sqrt{\frac{m_\mu m_\tau}{m_c m_t}} m_b V_{cb}^* V_{tb}^* & \frac{m_b}{m_t} m_\tau (V_{tb}^*)^2 \end{pmatrix} \\ \simeq & U_2 \text{diag} \left\{ \frac{m_d}{m_u} m_e (V_{ud}^*)^2, \frac{m_s}{m_c} m_\mu (V_{cs}^*)^2, \frac{m_b}{m_t} m_\tau (V_{tb}^*)^2 \right\} U_2^T, \end{aligned} \quad (30)$$

so that

$$\begin{aligned} m_\nu &= 2 \frac{v_L}{v_R} \frac{c_\nu}{c_e} (U_1 U_e U_2) \text{diag} \left\{ \frac{m_d}{m_u} m_e (V_{ud}^*)^2, \right. \\ & \left. \frac{m_s}{m_c} m_\mu (V_{cs}^*)^2, \frac{m_b}{m_t} m_\tau (V_{tb}^*)^2 \right\} (U_1 U_e U_2)^T. \end{aligned} \quad (31)$$

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13] leptonic mixing matrix is then given by

$$U_{\text{PMNS}} = U_e^\dagger U_\nu = U_e^\dagger U_1 U_e U_2. \quad (32)$$

For the given U_{PMNS} and U_2 , we can choose an arbitrary unitary U_e to determine the Yukawa couplings y_N and the Majorana masses m_N by inserting the unitary $U_1 = U_e U_{\text{PMNS}} U_2^\dagger U_e$ and the known \tilde{m}_ν [cf. (22)] to Eq. (27). The seesaw and perturbative conditions can be satisfied in a wide parameter space of y_N and m_N . Clearly, the neutrino mass matrix (31) can perfectly produce the mass spectrum (5) for generating the relation (2). Compared with Eqs. (5) and (8), we can find

$$\epsilon = 2 \frac{v_L}{v_R} \frac{c_\nu}{c_e} = 1.60 \times 10^{-9}, \quad (33)$$

to determine the left-right symmetry breaking scale,

$$v_R = 2 \frac{v_L}{\epsilon} \frac{c_\nu}{c_e} = 2.18 \times 10^{11} \text{ GeV for } \frac{c_\nu}{c_e} = 1. \quad (34)$$

In summary we found the intriguing relation between the neutrino and charged fermion masses, $|m_{\nu_3}^2 - m_{\nu_1}^2| : (m_{\nu_2}^2 - m_{\nu_1}^2) :: V_{tb}^4 m_\tau^2 m_b^2 / m_t^2 : V_{cs}^4 m_\mu^2 m_s^2 / m_c^2$. We then proposed a left-right symmetric model to naturally explain this phenomenon. In our model, the normal hierarchy neutrino masses are fully determined by the charged fermion masses for a given left-right symmetry breaking scale. In turn, the left-right symmetry breaking scale is fixed by the observed neutrino masses. The predicted neutrino spectrum is possible to test in the future. Moreover, the leptoquark singlet scalar in our model is flexible to be at an accessible scale so that it can have some interesting implications on the present and future experiments.

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